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Revisiting Trilateration Method Based on Time-of-Flight Measurements for Navigation

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Abstract

This paper revisits trilateration in three-dimensional positioning. Specifically, range between a positioning target and the reference points through time-of-flight (ToF) measurements. In a ToF, range is yielded by multiplying the time required by a wave to travel between two points and its propagation speed. Position of the target can be then estimated once the number of references are adequate, i.e. at least three for two-dimensional positioning and four for three-dimensional one. In this paper, the positioning is considered for navigation where the target moves following a trajectory whilst the ToFs take place in a certain period. The target position at the time is computed based on the ToFs through least square estimation. Through a numerical simulation, it is shown that the trilateration can track a target's trajectory despite the decreasing performance at the end of the course.

Keywords: *Navigation, Pseudorange, Time-of-Flight, Trilateration*

Abstrak

Makalah ini meninjau kembali trilaterasi dalam posisi tiga dimensi. Secara khusus, rentang antara target posisi dan titik referensi melalui pengukuran waktu penerbangan (ToF). Dalam ToF, jangkauan dihasilkan dengan mengalikan waktu yang dibutuhkan gelombang untuk bergerak antara dua titik dan kecepatan rambatnya. Posisi target kemudian dapat diperkirakan setelah jumlah referensi memadai, yaitu minimal tiga untuk penentuan posisi dua dimensi dan empat untuk penentuan posisi tiga dimensi. Dalam tulisan ini, positioning dipertimbangkan untuk navigasi dimana target bergerak mengikuti suatu lintasan sementara ToF berlangsung dalam periode tertentu. Posisi target pada saat itu dihitung berdasarkan ToF melalui estimasi kuadrat terkecil. Melalui simulasi numerik, terlihat bahwa trilaterasi dapat melacak lintasan suatu target meskipun kinerjanya menurun di akhir lintasan.

Kata kunci: Navigasi, *Pseudorange*, *Time-of-Flight*, Trilaterasi

Introduction

Trilateration is a mathematical method of estimating position of a target based on its ranges toward several references. Although its recent applications are mostly known in the area of navigation and localization, e.g. aerospace (Molesky & Wilhelm, 2002), indoors (Andò et al., 2021; Marasigan, 2020), and underwater (Wetter & Seto, 2022); trilateration is also found to be useful in other area such as biomedical engineering (Alshareef et. al (2020); Blaiech et al, 2021). Differing from Thomas & Ros (2005), the use of term trilateration here is not limited to positioning problem that uses precisely three references.

In practice, a range used in trilateration is obtained by multiplying time required by a wave to travel between two points and its propagation speed, known as time-of-flight (ToF) (Bensky, 2016, p. 2). Therefore, a ToF measurement mainly deals with recording transmission and receiving timestamps of a wave in question. On the other hand, common types of wave deployed for the measurements are electromagnetic (Fang, 1985; Plets, 2019) and acoustic ones (Eustice et al, 2011).

In this paper, trilateration based on ToF measurements is revisited. Its contribution is to address computation steps from ToF measurements to positioning estimation. The steps are elaborated in the sense of tracking problem, where the target's position changes from a sequence to another. As a ToF measurement is inherently a discrete-time process, the dynamics of the positioning is also represented in a discrete form with a certain sampling period.

To maintain brevity, formulations in this paper follows an ideal ToF criterion, i.e. during the measurements the reference and target remain still; all clocks involved in recording timestamps are synchronous; and the wave propagates in a straight line–line of sight (LoS) (Simamora et al., 2019). Reader interested on the inclusion of uncertainties in trilateration may consult, e.g. Yang et al. (2020) and Pakanon et al. (2020) for indoor situations and the latter also for non-LoS condition, Simamora et al. (2022) for wave's bending trajectory, and Teoman & Ovatman (2019) for uncertain references.

Problem Statement

Navigation Scenario

Here, a positioning system with L fixed-references and $1/\tau$ update-rate is to be considered. The position of reference ($j=1,\dots,L$) is assumed to be fixed and known, i.e.

$\mathbf{r}_j = \begin{bmatrix} x_j & y_j & z_j \end{bmatrix}^T$. At sequence k^{th}, a target enters the system and receives signal waves from the references.

An illustration for of the above scenario for $L=5$ is shown in Figure 1. Through trilateration, position of the target is at $\mathbf{p}(k)$ can be estimated based on the range between the target and reference *j*, i.e. $d_j(k)$, ($j=1,\ldots,L$).

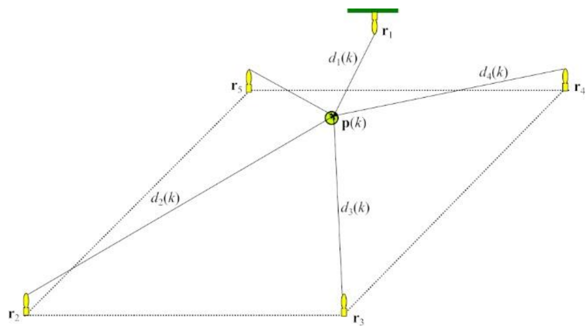

Figure 1. ToF measurements between five references and a navigation target.

Navigation Scenario

In Figure 1, range $d_j(k)$ is a geodesic distance, i.e. the shortest path– between reference and the target at k^{th}. This can be yielded obtained through a time-of-flight measurement at k^{th}, i.e.

$$d_j(k) = c \cdot \left[t_j(k) - t_{0j}(k) \right], \qquad (1)$$

where c denotes the wave's propagation speed while $t_{0j}(k)$ and $t_j(k)$ are its timestamps when being transmitted from reference j and received by the target, respectively. As reported by Eustice et al. (2011) and Webster et al. (2012), the technology to encode information about sender's position, sending time etc. to transmitted wave has been available.

As mentioned earlier, position of the target at k^{th}, i.e. $\mathbf{p}(k) = \begin{bmatrix} x(k) & y(k) & z(k) \end{bmatrix}^T$ can be estimated using trilateration method when there are $L \geq 4$ ToF measurements at k^{th}. Assuming that the ToF measurements are unbiased, (1) would be equivalent to

$$d_j(k) = \left\| \mathbf{p}(k) - \mathbf{r}_j \right\|, \qquad (2)$$

where

$$d_j(k) = \left\{ \left[x(k) - x_j \right]^2 + \left[y(k) - y_j \right]^2 + \left[z(k) - z_j \right]^2 \right\}^{\frac{1}{2}}.$$

It is of interest to obtain a more explicit expression than (2) in terms of relation between $d_j(k)$ and $\mathbf{p}(k)$. This could be achieved through the so-called pseudorange difference method. The method is proposed by Caffery (2000) and elaborated further by Batista (2015). Essentially, it is a sequence of algebraic manipulations towards (2) for two different references/ToF measurements. The method is applied as follows. Squaring both sides of (2) results in

$$d_j^2(k) = \mathbf{p}^2(k) + \mathbf{r}_j^2 - 2\cdot\mathbf{r}_j^T\cdot\mathbf{p}(k), \tag{3}$$

noticing that $\mathbf{p}(k)$ and $\mathbf{r}_j(k)$ are both 3×1 column vectors, hence

$$\mathbf{p}^T(k)\cdot\mathbf{r}_j(k) \equiv \mathbf{r}_j^T(k)\cdot\mathbf{p}(k).$$

Furthermore, taking (3) for reference i gives

$$d_i^2(k) = \mathbf{p}^2(k) + \mathbf{r}_i^2 - 2\cdot\mathbf{r}_i^T\cdot\mathbf{p}(k), \tag{4}$$

where $i = (1,\ldots,L)$ but $i \neq j$. Subtracting (4) from (3) yields

$$d_i^2(k) - d_j^2(k) = \mathbf{r}_i^2 - \mathbf{r}_j^2 - 2\cdot\left(\mathbf{r}_i^T - \mathbf{r}_j^T\right)\cdot\mathbf{p}(k), \tag{5}$$

where the term $\mathbf{p}^2(k)$ is now canceled. Rearranging terms in (4) while recalling that $\left(\mathbf{r}_i^T - \mathbf{r}_j^T\right) = \left(\mathbf{r}_i - \mathbf{r}_j\right)^T$, $\mathbf{r}_i^2 = \mathbf{r}_i^T\cdot\mathbf{r}_i$ etc. gives

$$\left(\mathbf{r}_i - \mathbf{r}_j\right)^T\cdot\mathbf{p}(k) = \frac{1}{2}\left[d_j^2(k) - d_i^2(k) + \mathbf{r}_i^T\cdot\mathbf{r}_i - \mathbf{r}_j^T\cdot\mathbf{r}_j\right] \tag{6}$$

For a given L, Eq. (5) will be specified for $U = L(L-1)/2$ equations (Batista, 2015), i.e.

$$\left.\begin{aligned}
\left(\mathbf{r}_1 - \mathbf{r}_2\right)^T\cdot\mathbf{p}(k) &= \frac{1}{2}\left[d_2^2(k) - d_1^2(k) + \mathbf{r}_1^T\cdot\mathbf{r}_1 - \mathbf{r}_2^T\cdot\mathbf{r}_2\right] \\
\left(\mathbf{r}_1 - \mathbf{r}_3\right)^T\cdot\mathbf{p}(k) &= \frac{1}{2}\left[d_3^2(k) - d_1^2(k) + \mathbf{r}_1^T\cdot\mathbf{r}_1 - \mathbf{r}_3^T\cdot\mathbf{r}_3\right] \\
&\vdots \\
\left(\mathbf{r}_{L-1} - \mathbf{r}_L\right)^T\cdot\mathbf{p}(k) &= \frac{1}{2}\left[d_{L-1}^2(k) - d_L^2(k) + \mathbf{r}_{L-1}^T\cdot\mathbf{r}_{L-1} - \mathbf{r}_L^T\cdot\mathbf{r}_L\right]
\end{aligned}\right\}, \tag{7}$$

where U is the number of possible combinations of pseudorange differences constituted by L references.

One may notice that the structure of Eq. (7) fits to the matrix-vector form of a system of linear equations, i.e.

$$\mathbf{A}\cdot\mathbf{p}(k) = \mathbf{b}(k), \tag{8}$$

where

$$\mathbf{A} := \left[\left(\mathbf{r}_1 - \mathbf{r}_2\right)^T \quad \cdots \quad \left(\mathbf{r}_{L-1} - \mathbf{r}_L\right)^T\right]^T$$

and

$$\mathbf{b}(k) := \frac{1}{2}\begin{bmatrix} d_2^2(k) - d_1^2(k) + \mathbf{r}_1^T \cdot \mathbf{r}_1 - \mathbf{r}_2^T \cdot \mathbf{r}_2 \\ \vdots \\ d_L^2(k) - d_{L-1}^2(k) + \mathbf{r}_{(L-1)}^T \cdot \mathbf{r}_{(L-1)} - \mathbf{r}_L^T \cdot \mathbf{r}_L \end{bmatrix}.$$

Therefore, $\mathbf{p}(k)$ can be estimated by finding the least square solution for (8) (Strang, 2016, p. 219), i.e.

$$\hat{\mathbf{p}}(k) = \left[\mathbf{A}^T \cdot \mathbf{A}\right]^{-1} \cdot \left[\mathbf{A}^T \cdot \mathbf{b}(k)\right], \tag{9}$$

where $\hat{\mathbf{p}}(k)$ denotes the estimated value of $\mathbf{p}(k)$.

Results and Discussion

To demonstrate the trilateration in (9) a numerical simulation is to be set up as follows. First, the reference system is configured as in Figure 1, whilst the numerical values for the reference position are listed in Table 1. The system is expected to tracking a target that moves in a trajectory from [550 5 5]^T m to [550 5 108]^T m as shown in Figure 1. In this scenario, the ToFs measurements take place for 1000 sequence with τ =1 s. Meanwhile, c is set to 1500m/s, i.e. a typical propagation speed of underwater acoustics.

Table 1. References' numerical values.

References	Position [m]
$\mathbf{r}_1$	$[600 \quad 600 \quad 2]^T$
$\mathbf{r}_2$	$[0 \quad 0 \quad 125]^T$
$\mathbf{r}_3$	$[1200 \quad 0 \quad 125]^T$
$\mathbf{r}_4$	$[1200 \quad 1200 \quad 126]^T$
$\mathbf{r}_5$	$[0 \quad 1200 \quad 123]^T$

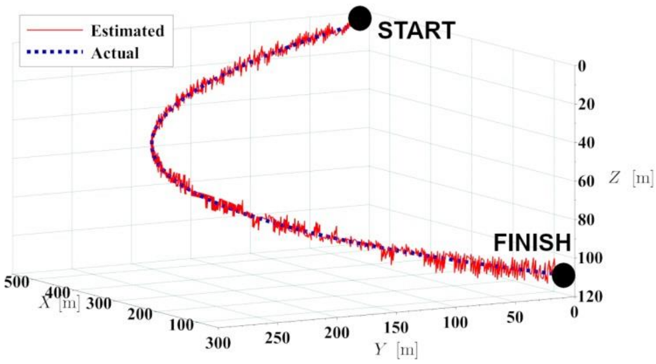

Figure 2. Trilateration on a tracking problem

As shown in Fig. 2, the reference system can track the target movement. This tracking using (9) based on ToF measurements shown in Figure 3.

To put it in a perspective, every 10^{-1} second of the ToFs in Figure 2 is equivalent to 150 m range measurement. As addressed by Batista (2015), this values means that small inaccuracies in recording timestamps, i.e. $t_{0j}(k)$ and $t_j(k)$ will affect a significant range bias. Such bias would be much larger in a reference system that based on electromagnetic wave, since for these waves $c = 3\times10^8$ m/s.

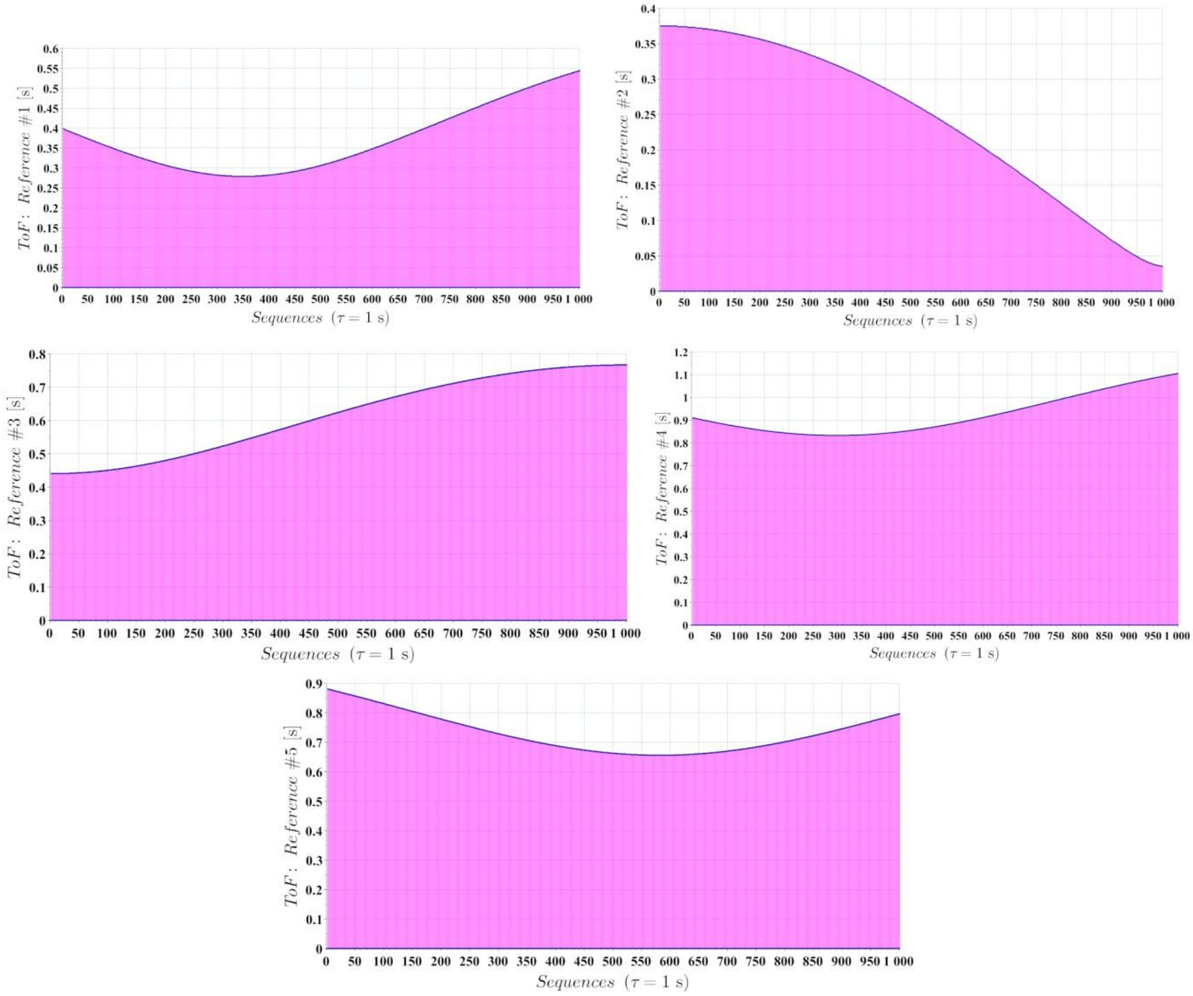

Figure 3. ToF measurements between the target and each reference

Meanwhile, estimation errors of the tracking are shown in Fig. 4. It can be seen that the estimation errors at XY axes tend to increase at the end of the course, whilst the error at the Z axis tend to converge. One possible mitigation to such dynamics is to deploy a Kalman Filter (KF) (Terajanu, 2013) where the estimator would rely more on the target's kinematics model when measurements are less reliable. In such case, the trilateration output would provide measurements in the KF's update state.

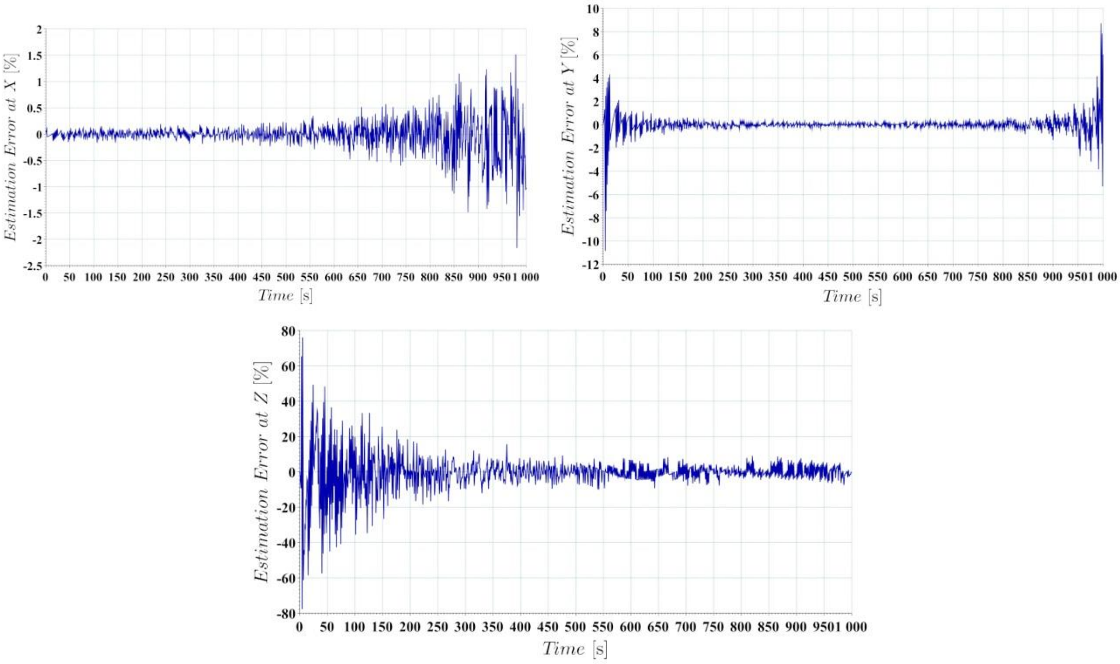

Figure 4. Estimation errors in *XYZ* axes.

Conclusion

Trilateration for navigation based on time-of-flight (ToF) measurements has been revisited. The trilateration was represented in a discrete form to comply with the discrete nature of ToF measurements. Through a numerical simulation, it was shown that trilateration could track a target trajectory despite the decreased performance at the end of the course.

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